

Euclidean Geometry and the *A Priori*

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ABSTRACT

Euclidean geometry is now commonly considered to have been justified *a priori* for Kant, but not for us. This entails that *a priori* justification is empirically defeasible. In this paper I will re-evaluate the status of alternative geometries such as Euclidean geometry and suggest that they are still justified *a priori*. What we need to change is our conception of *apriority*. The suggestion that will be offered is that we should move away from the epistemic discussion concerning *a priori* justification, as this leaves a more fundamental problem unanswered: the relationship between the *apriority* and the *truth* of a proposition. By distinguishing between pure geometry which can be understood as being *true in a model*, and applied geometry which is *true in the world*, we can redefine *apriority* in such a way that empirical indefeasibility can be maintained.

1. Introduction

Euclidean geometry is no doubt the most discussed example of *a priori* knowledge, notoriously so as it turned out not to even be true, much less necessary, contrary to what Kant famously thought. Kant considered Euclidean geometry to be an example of synthetic *a priori* knowledge, but this conception was soon undermined by so called non-Euclidean geometries, such as Lobachevski's and Riemann's, thus apparently making the question of which geometry is the actual or true geometry an empirical one. A natural conclusion is that geometry is not an *a priori* discipline after all, or, since it now appears that the actual geometry is non-Euclidean, at least Euclidean geometry is not *a priori*. However, this does seem satisfactory, for Euclidean geometry certainly appears to have been justified *a priori*. Something has to give: either we have to explain why Euclidean geometry was not justified *a priori* after all, or we have to acknowledge that *a priori* justification can be defeated by empirical evidence.¹

Among contemporary philosophers working on the *a priori*, the latter option has enjoyed much more support (cf. Russell 2008, BonJour 1998, Casullo 2003, Peacocke 2004). Indeed, perhaps the most important philosophical ramification of the discovery of non-Euclidean geometries was that *a priori* justification and knowledge are considered not just fallible, but empirically defeasible.² For instance, Albert Casullo's preferred analysis of *a priori* justification states that 'S's belief that *p* is justified a priori if and only if S's belief that *p* is nonexperientially justified' (2003: 33). Casullo does consider a stronger version of this analysis, which includes the requirement that the belief cannot be defeated by experience, but dismisses it as inadequate (ibid., 47). The major reason for the success of the weaker type of analysis that Casullo prefers is

1 I will not consider the view that all *a priori* knowledge must be analytic and necessary, in the context of this paper it will be assumed that there is something like synthetic *a priori* knowledge.

2 It is worth noting that Philip Kitcher's (1984) influential critique of *a priori* knowledge takes it to be empirically *indefeasible* – something that BonJour, Casullo and Peacocke contest.

presumably exactly the challenge that cases such as the one of Euclidean geometry introduce: *a priori* justification would seem to be exceedingly rare if not impossible if we maintain the requirement for empirical indefeasibility. In this paper I wish to question this ramification and defend the view that although *a priori* justification is indeed fallible, it is nevertheless empirically indefeasible. The outline of the argument that I will present goes as follows:

1. *A priori* justification (and knowledge) is empirically indefeasible.
2. Cases like Euclidean geometry appear to suggest that this is not the case, hence the commonly accepted conception of *a priori* justification takes it to be empirically defeasible.
3. I want to keep (1), so (2) needs to be addressed somehow.
4. To do this, we must distinguish between the *apriority* of a proposition and the truth of a proposition.
5. Even though the judgements that we make concerning the truth of a proposition are empirically defeasible, *a priori* justification is not.
6. *A priori* justification concerns the *metaphysical possibility* of the proposition.

I will first outline the problem of Euclidean geometry, and I will then analyse the status of alternative geometries. After this we can look closer into the nature of the axioms underlying geometry, which I will use as a basis to motivate a novel understanding of *a priori* justification and knowledge – an understanding which maintains the traditional requirement for empirical indefeasibility and suggests that *a priori* justification concerns metaphysical possibility.

2. The problem of Euclidean geometry

It is clear that even if the question concerning the geometry of the physical world is an empirical one, it does not entail that Euclidean geometry was not justified *a priori*, or indeed that geometry is not an *a priori* discipline. Instead, we might say that there are two different senses of geometry: geometry as a pure mathematical theory based on certain axioms, and geometry applied to the world. In fact, we might say that there are two senses in which a geometry can be true. Firstly, a geometry can be consistent in the sense that it is axiomatisable and all the true propositions of the theory can be derived from a set of basic axioms; we might call this *truth in a model*.³ Secondly, a geometry can be true after it has been interpreted and applied to the actual physical world, that is, it can be an accurate model of the physical world; we might call this *truth in the world*.

The story is not quite as simple as this, but it will serve for the purposes of setting up the discussion that will follow. It should be noted that the distinction between pure and applied geometry is something that Kant failed to make, in fact, a common critique of Kant's analysis of Euclidean geometry is based on his failure to make this distinction. However, as Michael Friedman (1985) has argued, this objection is not entirely fair to Kant: it is based on a modern understanding of logic, which was simply not available to Kant. I am not interested in Kant exegesis though. The question that I am interested in is what sort of implications does the apparent distinction between pure and applied

3 For our purposes, 'truth in a model' is best interpreted in the lines of John Etchemendy (1990: 24), namely that a sentence is true in a model if and only if it would have been true had the model been an accurate description of the world. A minimal requirement for 'truth in a model' is that the theorems of the theory in question follow from its axioms and postulates. However, although this is necessary for 'truth in a model', it is not sufficient: a theory can be sound in this sense, but still fail to satisfy Etchemendy's criterion, as there can be consistent models that are not genuinely possible. We will return to the details of this later, the modal content in the definition of a model turns out to be of crucial importance.

geometry have for the notion of *apriority*. The topic has of course received an enormous amount of attention, but I believe that the full implications of the story concerning Euclidean geometry are yet to be cashed out. Ultimately I think that we must revise our understanding of *apriority* quite fundamentally, but here I will primarily attempt to outline the problem and motivate further discussion.⁴ Accordingly, although nothing strikingly novel will be said about Euclidean geometry itself, the analysis suggests that we can still accommodate a notion of *apriority* which is empirically indefeasible. Whether this conception is closer to the Kantian one or not is another question, and one that we will not attempt to settle, but, at any rate, this conception is closer to the traditional debate concerning the *a priori*. Indeed, as Hartry Field puts it:

Some writers use ‘a priori’ in a way that imposes no empirical indefeasibility requirement, but it seems to me that that removes the main philosophical interest of apriority: traditional debates about the apriority of logic and Euclidean geometry have largely concerned the issue of whether any empirical evidence could count against them. (Field 2000: 117.)

Field’s own suggestion is to conceive of the *a priori* as an evaluative notion, but I will not examine his approach here. I simply wish to emphasise the importance of the problem concerning empirical defeasibility and suggest an alternative solution.

I mentioned above that even if the question of actual geometry is an empirical one, that does not mean that Euclidean geometry could not have been justified *a priori*. Indeed, philosophers like Laurence Bonjour, Albert Casullo and Christopher Peacocke would perhaps all agree about this. This suggests an obvious line of thought: *apriority*, the epistemic notion that it is, ought to be defined in terms of epistemic warrant, that is, if one’s belief in something is justified *a priori* and it is true, then we are dealing with *a*

⁴ I have discussed the topic before in (Tahko 2008), where I first sketched a new definition of *a priori* knowledge in terms of metaphysical modality.

priori knowledge. Of course, by this definition, Euclidean geometry is not *a priori* knowledge, since it is not true. However, the requirement for truth could perhaps be modified, or even removed altogether, so that every belief that is justified *a priori* constitutes *a priori* knowledge, regardless of its truth in the actual world. In fact, as we will see, distinguishing ‘truth in a model’ and ‘truth in the world’ might enable us to do something like this. Be that as it may, the details of *a priori* justification are not the primary topic of this paper. In Casullo’s terms, I am more interested in a *nonreductive* rather than a *reductive* approach to *a priori* knowledge – the first is concerned with the analysis of the concept of *a priori* knowledge, while the latter is concerned with *a priori* justification (Casullo 2003: 10; 2009: 78). Moreover, if we give a story about Euclidean geometry strictly in terms of *a priori* justification, this still leaves open the question concerning truth, specifically, since Euclidean geometry turned out not to be true, why should we still consider it to be *a priori*? It may be the case that, for Kant at least, Euclidean geometry was justified *a priori*, but presumably it is not justified *a priori* for us.

Accordingly, what follows is a study of the metaphysical rather than the epistemic status of Euclidean geometry. I believe that this is appropriate even though *apriority* is arguably an epistemic notion, since I take it that *truth* is a metaphysical notion, and the central question of this paper concerns the role of truth in this picture. As it will turn out, the truth of a proposition is not a necessary requirement for the *apriority* of that proposition. But enough of these preliminaries, I hope that the reasons for my approach will become clear during the course of the paper.

To begin with, a brief recap of the emergence of non-Euclidean geometries might be in order.⁵ Euclidean geometry is based on five axioms and five postulates; it is postulate five or the so called parallel postulate which is at the core of Euclidean geometry. Given

5 I rely on Lawrence Sklar’s (1976) classic presentation of the topic here.

the other axioms and postulates, the parallel postulate can be expressed in a concise manner: ‘through a point outside a given line one and only one line can be drawn which does not intersect the given line, no matter how far it is extended’ (cf. Sklar 1976: 15). The parallel postulate, however, is not quite as self-evident as the other axioms and postulates, and despite attempts to derive it from them, it was only a matter of time before someone decided to replace the problematic postulate with another one. This is exactly what mathematicians like Gauss and Lobachevski did, independently and almost simultaneously. They replaced the parallel postulate with one that allows for more than two non-intersecting lines to be drawn. Similarly, with only a small change in the interpretation of one of the other postulates and by replacing the parallel postulate with one that does not allow for any parallels, Riemann came up with another alternative non-Euclidean geometry – one that serves the purposes of contemporary physics considerably better. What is important for us is that all of these alternative geometries are *consistent*. This simply means that no contradictions follow from the set of the basic premises of the theories. We might add a requirement for completeness, i.e. that the theorems of the theory must follow from its axioms and postulates. The consistency of Lobachevskian and Riemannian geometries has been demonstrated, but we need not concern ourselves with the proofs.

It should perhaps be noted that Euclidean geometry in its original form is not complete, as there are certain hidden assumptions that are required for the purposes of deriving all the theorems of Euclidean geometry. So in its original form Euclidean geometry did not satisfy the requirement that was added above. However, Hilbert’s work in the end of the 19th century provided a fully axiomatised and complete version of Euclidean geometry. For the purposes of simplicity I will refer to Hilbert’s version when I talk about Euclidean geometry. What makes Hilbert’s work particularly important is that, given his version of Euclidean geometry, the role of the parallel

postulate, which Hilbert called the Axiom of Parallels, becomes even clearer: we can see that simply by changing this axiom a number of alternative geometries can be formulated. The question that I will now turn to concerns the status of these possible, alternative geometries.

3. The status of alternative geometries

It is now well known that General Relativity seems to have settled the question of which geometry is the actual one in favour of Riemannian geometry. However, it is not entirely clear what kind of implications this has for the status of alternative geometries. For one thing, although General Relativity has passed all empirical tests, there still remains at least a philosophical concern that all the empirical data that supports the existence of a curved spacetime and a corresponding gravitational field, as postulated by Einstein's General Relativity, is also compatible with a flat Minkowski spacetime, which fits the Euclidean picture. From an empirical point of view the idea may seem ludicrous, as the reconciliation of a flat Minkowski spacetime with the apparent curvature of the actual spacetime requires postulating yet unknown forces of a rather peculiar type. This scenario is familiar from Henri Poincaré's (1905) classic conventionalist account, where he demonstrates that, given suitable laws of physics, a group of two-dimensional beings living on a closed two-dimensional Euclidean disk could be fooled in such a way that they would consider themselves to be living on a Lobachevskian plane of infinite extent. The trick is to provide these beings with measuring devices such as rigid rods that actually change their length in regard to their position on the disk. Now, it seems that these beings would have no empirical means to decide what the true nature of their world is. The next step is to insist that something similar could be happening in our world, and thus it becomes merely a matter of convention which geometry we wish to adopt. This is because our empirical information

is compatible with all the alternatives if appropriate changes in the laws of physics are postulated.

We need not engage with Poincaré's scenario in detail, although the discussion that followed and especially Eddington's and Reichenbach's replies are very interesting.⁶ However, it is perhaps worth noting that the risk of collapsing into utter scepticism is very real here; Sklar (1976) lists a number of positions that we could take regarding Poincaré's problem, but as Laurence Bonjour (1998: Appendix) has observed, the sceptical conclusion is very difficult to avoid if we go with Poincaré. It may be useful to keep this in mind, but in what follows we will assume that nothing quite as radical as Poincaré's scenario is true of our world; if for no other reason then simply because radical conventionalism is a rather unfruitful position. Still, a number of somewhat more plausible concerns remain, most notably the incompleteness of the current picture: despite the accuracy with which General Relativity fits the empirical data, it is not fully consistent with the standard model of quantum mechanics. Accordingly, the story will not be complete until we have a theory of quantum gravity which enables us to reconcile General Relativity with quantum mechanics.

So, although it would appear that Riemannian geometry can claim the title of *true in the world*, the situation is certainly quite complicated. We should now look into the status of alternative geometries in more detail, and especially into the distinction between *pure* and *applied* geometry that was briefly introduced above. A natural way to cash out the distinction is to say that pure geometry is *a priori* and applied geometry is *a posteriori*, as the first concerns pure mathematical modelling and the latter concerns the actual geometry of the physical world. If this distinction is valid, then it would seem that we have a clear case for the epistemic status of Euclidean geometry: it is *a priori* insofar as we consider it as a sample of pure geometry. More generally, any fully

⁶ See Sklar (1976) for an extensive discussion.

axiomatisable alternative geometry considered as pure geometry is *a priori*. In this sense, Riemannian geometry as well is *a priori*, but it would also appear to be true in the applied sense. Now, due to the empirical defeasibility of applied geometry, it could always turn out that the geometry we considered to be actual is in fact a merely *possible*, alternative geometry, but this does not threaten the *apriority* of geometry; we already knew that the structure of the physical world is an *a posteriori* matter.

I suggested before that we might talk about two different senses of *truth* in this connection: *truth in a model* and *truth in the world*.⁷ Some might find this misleading, but I do not think that it is any more misleading than the distinction between *logical truth* and the truth of a theory. For our purposes, logical truth may be understood in terms of truth in a model, but this of course still requires an explanation concerning the interpretation of models. Here I am sympathetic to an interpretation familiar from John Etchemendy (1990: 24), namely that a sentence is *true in a model* if and only if it would have been true had the model been an accurate description of the world. This is not the place for a full analysis of the notion of logical truth, but what is crucial for the task at hand is that on this interpretation of ‘model’, it is required that the model *could have been true*, that is, it must be the case that the world could have turned out to be like the model depicts – otherwise we are not dealing with a valid, possible model. So the concept of a model that we have here takes models to be *possible* representations of the world. The nature of the modality at hand is a question that we will return to, but for now it is sufficient to note that *truth in a model* is independent of any empirical considerations; all that is required is that the model is *genuinely* possible, to use Etchemendy’s terminology (ibid., 25). Contrary to this, the truth of a theory *in the world*, if it is a theory of the physical world, will be settled by empirical means. This

⁷ This contrasts with Sklar’s approach. He suggests that pure geometry does not concern truth at all, as logical forms are not true in any way, but I think that there is an entirely legitimate use of *truth* here: this is what the notion of *logical truth* is for (cf. Sklar 1976: 104-108).

sense of truth is of course contingent and subject to revision, but the previous sense of truth is, in at least some sense, necessary, albeit only relative to a particular model.

We must be careful here, for logical truths come in many forms, or, more accurately, in many models. For instance, classical and paraconsistent logics (cf. Priest 2006) are of course mutually exclusive and very different things are true in their respective models. Accordingly, any necessity associated with logical truth will be restricted to a particular model. We will return to this example below. I should add here that it is an entirely different question whether there is a logical system that can be applied to the physical world and which accurately depicts it – we have to distinguish between pure and applied logic just as we have to distinguish between pure and applied geometry. Those who are unsure about using the notion of truth in this way may interpret it as follows: the only real sense of *truth* is, say, correspondence to physical reality, and Euclidean geometry, for instance, is at best *consistent*. This move is entirely possible and nothing that I will say depends on this issue very heavily, but it seems to me that this would also be bad news for the notion of *logical truth*, as it most certainly does not concern physical reality, but rather possible models of reality. This is one reason why I prefer to simply use the notion of truth in two different senses.

So, does the distinction between pure and applied geometry understood in the sense of *truth in a model* and *truth in the world* enable us to save the Kantian conception of *a priori* knowledge? The consensus, I believe, is that more radical changes are needed. Indeed, as Penelope Maddy (2000: 102) has put it, ‘[t]here is no particularly Kantian way’ of dealing with the problems introduced by General Relativity, non-Euclidean geometries and quantum mechanics.⁸ Even those who consider themselves to be neo-Kantians, such as Michael Friedman (e.g. 2000), have a radically different conception of the *a priori*. Friedman, sympathetic to Reichenbach’s (1920) early suggestion,

⁸ See Friedman (1985) for more discussion on Kant’s conception of the *a priori*.

defends the idea that *apriority* is historically relative and dynamic: all that is maintained are certain constitutive elements which govern scientific, mathematical-physical inquiry. Hence, we must abandon the idea of a necessary, empirically indefeasible *a priori* and adopt only the Kantian idea of conceptual elements that are not contained in pure empirical perception, that is, ‘the object of knowledge is not immediately given but constructed’ (Reichenbach 1920: 49). The downside of this relativised conception of the *a priori* is that it appears to entail almost Kuhnian relativism. However, I will suggest that we do not need to go this far. Rather, we only need to abandon the requirement for necessity, whereas empirical indefeasibility can be maintained. As we will see, this produces a conception of the *a priori* which is at least in some sense closer to the traditional conception as well as immune to the problems produced by the case of Euclidean geometry.

A satisfactory resolution of the problems posed by alternative geometries is certainly required if we hope to have any such notion of *apriority*, and I think that we have the elements for that resolution in the distinction between pure and applied geometry, although we need to develop on it. There has been some discussion in the rationalist camp concerning this distinction, but at least Bonjour thinks that it is not entirely satisfactory; he thinks that there might be a further distinction to be made according to which geometry (and especially the parallel postulate) is neither pure nor applied to the physical world, but would instead be based on some sort of intuitive notion of straightness (1998: 223-224). While Bonjour remains neutral regarding the question of whether such intuitive notion of straightness indeed exists, it is clear that this would be something that a rationalist might find appealing. To me, this type of intuition sounds quite mysterious, but I do not think that we need to postulate the further distinction that Bonjour suggests to be able to salvage something of the traditional conception of the *a priori* while at the same time resolving the problems that alternative geometries

introduce.

The most important aspect of my suggestion concerns the modal status of these alternative geometries. I have already indicated that there is a sense in which alternative geometries, despite their mutual exclusiveness, may even be considered to be necessary. This is the sense of necessity that we also associate with logical truths. For instance, the law of non-contradiction is presumably necessary in some sense, but there are nevertheless fully consistent logical systems in which the law of non-contradiction does not hold, i.e. paraconsistent logics. But paraconsistent logics operate in a different model and hence we might say that the necessity of the law of non-contradiction is restricted to the model of classical logic.⁹ I will attempt to develop the analogy concerning alternative logics to motivate my suggested analysis of the status of alternative geometries.

There is a natural way to make the terminology at hand more rigorous, but it requires a distinction between the types of modality associated with the entities under investigation. We may represent the modality at hand by talking about *logical modality*, i.e. possibility and necessity strictly in virtue of the laws of logic. We may call this *strict logical modality* (Lowe 1998: 15). The space of logically possible worlds is presumably defined by what the laws of logic *are*, and since we have a number of mutually exclusive sets of logical laws, we can formulate a number of different sets of logically possible worlds and a corresponding notion of logical necessity for each set of worlds. However, if this route is taken, then it seems that all of these sets of logically possible worlds must also be possible in some more general sense. Indeed, this is presumably the sense of *genuine possibility* which Etchemendy (1990: 25) postulates as a requirement for all possible models. We could perhaps take this notion as unanalysed, as Etchemendy appears to do, but I believe that we have good reasons to think that the

⁹ See (Tahko 2009a) for more discussion about the status of the laws of logic.

only kind of modality that might qualify is *metaphysical possibility*: given our definition of truth in a model, we need genuine modality to reflect all possible configurations of the world and only metaphysical possibility fulfils this requirement.

This may seem alarming, as it is widely agreed that there are logical possibilities which are metaphysically impossible, such as the possibility of water failing to be H₂O, when ‘Water = H₂O’ is considered to be metaphysically necessary. However, if we interpret logical truth in the fashion that was suggested above, namely in terms of possible descriptions of the world, i.e. genuinely possible models, then it would appear to follow that logical possibility must be a proper subset of metaphysical possibility, that is, all logical possibilities are metaphysical possibilities. Incidentally, I think that the metaphysical necessity of ‘Water = H₂O’ is contentious at best, so it may still be the case that water failing to be H₂O is logically (and also metaphysically) possible.¹⁰

There is obviously much more to be said about the view concerning the status of logic suggested here, but that is something that can take place elsewhere, as I am only proposing an analogy here rather than trying to defend this view of logic. What we can take from this analogy is the following. If what has been said above is correct, then it follows, given the existence of alternative geometries and alternative logics, that neither a given geometry or a given logic can be *metaphysically necessary*. It may be that certain axioms or certain laws of logic are metaphysically necessary, but given that the alternative systems of logic and geometry include mutually exclusive axioms and laws, they cannot be metaphysically necessary.¹¹

10 See (Tahko 2009b) for further discussion about metaphysical necessities.

11 For the record, my preferred analysis of metaphysical modality is in terms of essences, following Kit Fine (1994), but defending this view here would take us too far from our original topic.

4. The status of the axioms

We now have the tools to inquire deeper into the status of alternative geometries. As we saw before, the difference between Euclidean, Riemannian and Lobachevskian geometries, for instance, can simply be cashed out in terms of one axiom – the one concerning how many parallels can be drawn. Since the other axioms are shared, it may even be the case that they are *metaphysically necessary*, that is, shared by all other alternative geometries.¹² Thus, the question, as we might have guessed, concerns the epistemic status of the axioms, and most importantly the axiom regarding parallels. But how do we come to know the axioms? According to Poincaré’s conventionalism, there is not much to say here: we may choose to believe in whatever axioms we please, they are simply a matter of convention. The empiricist will of course disagree and would hope that we do have some means, empirical means, to decide which axioms to believe in. Both of these answers, however, concern the status of axioms in *applied* geometry.

Surely, whichever geometry turns out to be the actual one, and even if we have no definite means to settle this, there is a story to be told about *how* we came up with the axioms in the first place. Of course, the axioms were supposed to be self-evident or intuitive, but given that we have *competing* intuitions, the self-evidentness of the axioms seems questionable. It might be worthwhile to note here that the emergence of non-Euclidean geometries was certainly not due to empirical evidence, but rather due to the fact that the parallel postulate did *not* appear to be quite as self-evident as the other axioms (Sklar 1976: 16 ff.). In fact, Euclid himself did probably not think that the parallel postulate is self-evident (Torretti 2010). As Lobachevski and others demonstrated, alternative, consistent geometries can be constructed by altering the

12 Although this might not be the case, as there are geometries, such as Riemann’s general analytic geometry of curved spaces, which are not presented in an axiomatised form and might even be non-axiomatisable.

parallel postulate. So it seems that we have good reasons to think that all these alternative geometries were the result of *a priori* work; this is something that others have insisted upon as well (e.g. Jones 1946: 143).

If it is agreed that the original axioms postulated by Euclid are in at least some sense *a priori*, perhaps even necessary apart from the parallel postulate – as at least most of the other axioms are also shared by all alternative geometries – then we have something to work with. It appears that Euclidean geometry is not the actual geometry, but I think that it is correct to say that it *could* have turned out to be the actual one. That is, Euclidean geometry is *true in a model*, as it would have been true of the world had the model been accurate. If this is correct, then we can say that Euclidean geometry is metaphysically possible, following the line suggested in the previous section. A similar idea can be seen in early work by Philip C. Jones: ‘Barring errors in deduction, therefore, all geometries are true and a priori’ (1946: 8). What is crucial here is the use of the word ‘true’: we must be dealing with truth in a model here. Accordingly, although *a priori* justification is fallible due to possible human errors in deduction, the *apriority* of geometry is independent of the truth of the geometry in the applied sense. All geometries that are metaphysically possible are *true in a model* and this is sufficient for *apriority*.

Now, although there is no doubt that we are much more interested in the *actual* geometry – the geometry that is *true in the world* – than merely possible geometries, the problem is that the question of the actual geometry, since it is a question of applied geometry, will always be an empirical one. Thus, due to the fallibility of empirical knowledge, it is at least conceivable, and quite plausibly also metaphysically possible, that our current understanding of the actual geometry of the physical world is mistaken, even without resorting to the extreme Poincaréan scenario. So it would seem that, however unlikely it may be, it is still metaphysically possible that further empirical

information will corroborate Euclidean geometry instead of Riemannian geometry.

By emphasising the empirical defeasibility of the *actual* geometry, I wish to highlight the peculiar status of alternative geometries: although it may seem that all but the actual geometry are in some sense futile, the problem is that we can never determine which geometry is the actual one with absolute certainty. Among other things, this justifies inquiry into pure geometry as opposed to applied geometry – indeed, without such inquiry, non-Euclidean geometries might never have emerged. Moreover, even if we are primarily interested in the actual geometry of the physical world, any geometry will first have to be formulated as a pure mathematical theory. We may proceed to test this theory by empirical means, and we of course must do this to determine whether it is the actual one, but the importance of pure geometry should not be underestimated. The structure of this process of inquiry can be illustrated with the help of the modal notions that were introduced above.

If we agree that alternative geometries are metaphysically possible, we may consider the study of pure geometry as a study of metaphysical possibilities. Ultimately, this inquiry aims to establish the actual geometry, but we must start by considering which geometries are possible. Also, given that empirical verification will be needed to establish the actuality of a given geometry, the process is divided. It seems that this division also marks the boundary between *a priori* and *a posteriori* inquiry: the study of possible geometries is *a priori* and the study of the actual geometry of the physical world is *a posteriori*. Accordingly, we could say that pure geometry is *a priori* inquiry into how things could possibly be, or, if you like, inquiry into the space of metaphysically possible worlds, one of which may be the actual world.

Is there a tension between the *apriority* of pure geometry and the *aposteriority* of applied geometry? I do not think so. It may have come as a surprise that there are alternative geometries, but now that we are aware of them, we may compare the

situation, for instance, to alternative accounts of quantum gravity, such as superstring theory and loop quantum gravity. They are alternative accounts of how to reconcile quantum mechanics and General Relativity, and only one of them – insofar as they are mutually exclusive and internally consistent – can be true in the sense that it accurately describes the phenomenon of gravity in the physical world. But if both theories are indeed internally consistent, then surely we must say that they are both *possible*. In this sense they are like alternative geometries: only one geometry can be actual but the others are still possible.

5. Towards a novel understanding of *apriority*

The view that I have attempted to motivate is that *a priori* justification and knowledge are empirically indefeasible. The case of Euclidean geometry does not compel us to abandon this classic criterion of *apriority*. It *does* require some revisions in our conception of *apriority*, but these revisions are arguably a less radical departure from the traditional notion of *a priori* knowledge than abandoning empirical indefeasibility.

We might compare this analysis with that of Bertrand Russell (1897), who suggested that Euclidean geometry may be *a priori* even if the parallel postulate itself is not. That is, we can divide the axioms of geometry into two classes: one of which is *a priori* and shared by both Euclidean and non-Euclidean geometries, and one which distinguishes Euclidean geometry from non-Euclidean alternatives and is empirical. However, while Russell's idea bears an important similarity to the one that we have been discussing, there is also a crucial difference: according to the current line of thought, the parallel postulate as well is *a priori* rather than *a posteriori*. It is indeed an empirical question whether the parallel postulate is true in the actual world, but *apriority* understood as suggested above concerns metaphysical possibility, and it appears that the parallel

postulate as well as the alternative axioms suggested by Gauss, Lobachevski and Riemann are all metaphysically possible in virtue of the consistency of these geometries. As Friedman has observed, this line of thought differs radically from that of Kant:

On the Russell-inspired interpretation [of Kant's conception of geometry] there can be no question of non-Euclidean geometries for Kant. Non-Euclidean straight lines, if such were possible, would have to at least possess the order properties – denseness and continuity – common to all lines, straight or curved. And, on the present interpretation, the only way to represent (the order properties of) a line – straight or curved – is by drawing or generating it in the space (and time) of pure intuition. But this space, for Kant, is necessarily Euclidean (on both interpretations). It follows that there is no way to draw, and thus no way to represent, a non- Euclidean straight line; and the very idea of a non-Euclidean geometry is quite impossible. (Friedman 1985: 488.)

The reason why a solution in the lines of what I have proposed would not have been available for Kant is that he did not have access to the type of model-theoretic tools that I have relied upon. I have suggested that we should view alternative geometries as metaphysically possible models, but for Kant there is no sense of possibility like this (ibid., 503). Only Euclidean geometry is possible in Kant's sense of possibility, which determines the possibility of a geometrical figure not in virtue of freedom from contradiction, but in virtue of the constraints that the formal conditions of experience, which all objects of experience must fall under, introduce.

So much for Kant, but why is Russell's solution not sufficient? Well, if we accept the distinction between pure and applied geometry, as it seems reasonable to do, then the original problem concerning empirical defeasibility re-emerges despite Russell's move. Specifically, since the epistemic status of the parallel postulate and other axioms of

geometry is the same when considered as pure geometrical principles – they are all *a priori* – we have no means to distinguish them in virtue of their epistemic status at this stage. The only way that we can distinguish between them is by applying them to the world, but this introduces an empirical element. It should also be noted that at the time when the alternatives to the parallel postulate were introduced, no empirical information suggested that one rather than the other would be true of the actual world. For a brief period of time in the beginning of the 20th century, all the alternative geometries were at the same line. Only when considered as applied geometry, and only after the empirical verification of Einstein’s work, were we able to determine that the parallel postulate differs from the other axioms of Euclidean geometry and the Riemannian alternative to the parallel postulate, namely, it is not true of the actual world.

It may be objected here that, in fact, the parallel postulate has always been considered less self-evident than the other axioms of Euclidean geometry and hence its *apriority* can be legitimately questioned. However, being self-evident can hardly be a necessary condition for *apriority*, as other, much more complex and certainly not self-evident mathematical truths are also often considered to be *a priori*. On the contrary, the fact that the *apriority* of the parallel postulate has been questioned only underlines the problem at hand: it is not clear what constitutes *a priori* knowledge. So why exactly would the parallel postulate not qualify as *a priori* knowledge? The only obvious answer seems to be because it is not true of the actual world.

The following dilemma emerges concerning the notion of *apriority*: either we have to include *truth in the world*, that is, actuality, among the criteria of *a priori* knowledge, or we must accept any consistent possible proposition as *a priori* regardless of its status concerning the actual world. But if we go for the first horn, then there is nothing that separates *a priori* knowledge from *a posteriori* knowledge, as it will always be an empirical question whether a given proposition is true in the world, save for any

metaphysically necessary propositions. If we further introduce metaphysical necessity as a criterion for *apriority* to save the definition, then the class of *a priori* propositions will be thin indeed, as even the most fundamental laws of logic may be empirically defeasible. Not only that, but all the classic problems concerning *a priori* knowledge re-emerge: how are we supposed to know that a certain proposition is metaphysically necessary when even the most celebrated example, Euclidean geometry, turned out not to qualify? The upshot would be that only analytic truths qualify as *a priori*. This would mean that the notion of *apriority* collapses into the notion of analyticity, which is certainly not desirable if we hope to salvage something of the original conception of *apriority*.

Accordingly, going for the first horn of the dilemma fails to give us a notion of *apriority* which would have much in common with the traditional conception, or indeed be very useful at all. Hence, I suggest that we should choose the second horn of the dilemma and accept that Euclidean geometry, including the parallel postulate, is still very much *a priori* and no empirical result can change that. The idea is simple enough and already familiar from the distinction between pure and applied geometry. However, we can apply this idea to all *a priori* propositions, not just to the ones concerning geometry. The underlying idea is that *apriority*, and *a priori* justification, concerns metaphysical possibility, the possible ways that the world might be, although we will not go any further into the details here.¹³ The implications for the notion of *apriority* are radical. The upshot is not simply that Kant was *a priori* justified to believe in Euclidean geometry, but that we are as well. There is no *metaphysical* necessity involved in the proposed account of *apriority*, but one important classic criterion for *apriority* is maintained: all *a priori* propositions are empirically indefeasible, and, if you like, timeless. This is where the proposed account differs from most contemporary accounts

¹³ See (Tahko 2008) for further discussion.

of *a priori* justification and knowledge. As was demonstrated earlier, we can even maintain that there is *some* sense of necessity involved, albeit this necessity is restricted to a given model. Thus, although this account implies that the class of *a priori* knowledge is radically inflated, it does give us clear criteria for *apriority*, and certainly fares better than either the traditional conception or most contemporary ones when faced with the challenge of empirical defeasibility.

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